

Unit 3: Product Rule and Quotient Rule

The Product Rule

$$(f \cdot g)'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

$$= F \cdot S' + F' \cdot S$$

Examples:

1. $f(x) = (x^4 + 1)(2x^2 - 1)$

$$f'(x) = (x^4 + 1)(4x) + (4x^3)(2x^2 - 1)$$

$$f'(x) = 4x^5 + 4x + 8x^5 - 4x^3$$

$$f'(x) = 12x^5 - 4x^3 + 4x$$

2. $h(x) = (3x^4 + 5x^2 - 3)(6x^3 - 7)$

$$h'(x) = (3x^4 + 5x^2 - 3)(18x^2) + (12x^3 + 10x)(6x^3 - 7)$$

$$h'(x) = 54x^6 + 90x^4 - 54x^2 + 72x^6 - 84x^3 + 60x^4 - 70x$$

$$h'(x) = 126x^6 + 150x^4 - 84x^3 - 54x^2 - 70x$$

3. $v(x) = (5x^7 - 9)(3x^5 + 8)$

$$v'(x) = (5x^7 - 9)(15x^4) + (35x^6)(3x^5 + 8)$$

$$v'(x) = 75x^{11} - 135x^4 + 105x^{11} + 280x^6$$

$$v'(x) = 180x^{11} + 280x^6 - 135x^4$$

The Quotient Rule

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

$$= \frac{B \cdot T' - T \cdot B'}{B^2}$$

Examples:

4.

$$k(x) = \frac{3x+1}{x^2-1}$$

$$k'(x) = \frac{(x^2-1)(3) - (3x+1)(2x)}{(x^2-1)^2}$$

$$k'(x) = \frac{3x^2 - 3 - 6x^2 - 2x}{x^4 - 2x^2 + 1}$$

$$k'(x) = \frac{-3x^2 - 2x - 3}{x^4 - 2x^2 + 1}$$

5. $f(x) = \frac{4x^3 - 5x + 3}{5x^2}$

$$f'(x) = \frac{5x^2(12x^2 - 5) - (4x^3 - 5x + 3) \cdot 10x}{(5x^2)^2}$$

$$f'(x) = \frac{60x^4 - 25x^2 - 40x^4 + 50x^2 - 30x}{25x^4}$$

$$f'(x) = \frac{20x^4 + 25x^2 - 30x}{25x^4} = \frac{5x(4x^3 + 5x - 6)}{25x^4}$$

$$f'(x) = \frac{4x^3 + 5x - 6}{5x^3}$$

Name: _____

$$6. f(x) = \frac{3x^2 - 5}{x^3 + 7}$$

$$f'(x) = \frac{(x^3 + 7)(6x) - (3x^2 - 5)(3x^2)}{(x^3 + 7)^2}$$

$$f'(x) = \frac{6x^4 + 42x - 9x^4 + 15x^2}{(x^3 + 7)^2} = \frac{-3x^4 + 15x^2 + 42x}{(x^3 + 7)^2}$$

$$f'(x) = \frac{-3x(x^3 - 5x - 14)}{(x^3 + 7)^2}$$

The Product Rule

$$(f \cdot g)'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$
$$= F \cdot S' + F' \cdot S$$

Examples:

1. $f(x) = (x^4 + 1)(2x^2 - 1)$

$$f'(x) =$$

2. $h(x) = (3x^4 + 5x^2 - 3)(6x^3 - 7)$

$$h'(x) =$$

3. $v(x) = (5x^7 - 9)(3x^5 + 8)$

$$v'(x) =$$

The Quotient Rule

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$
$$= \frac{B \cdot T' - T \cdot B'}{B^2}$$

Examples:

4. $k(x) = \frac{4x^3 - 5x + 3}{x^2}$

$$k'(x) =$$

5. $j(x) = \frac{3x + 1}{x^2 - 1}$

$$j'(x) =$$

6. $p(x) = \frac{3x^2 - 5}{x^3 + 7}$

$$p'(x) =$$